Evaluation of Surface vs Bulk Contributions in Sum-Frequency Vibrational Spectroscopy Using Reflection and Transmission Geometries†

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An experiment was designed to measure sum-frequency generation (SFG) spectra in both reflection and transmission. The result from an octadecyltrichlorosilane monolayer on a substrate reveals that while the resonant spectra in the two geometries both originate from the surface monolayer, the bulk substrate has a significant contribution to the nonresonant background in the transmitted spectra. Study of a polyethylene film also shows that the bulk contribution to the SFG spectra is only significant for the transmission geometry, exhibiting a resonant mode not observable in the reflected spectra.

I. Introduction

In recent years, surface vibrational spectroscopy via infrared—visible sum-frequency generation (SFG) has been developed into a powerful tool to study the structures of surfaces and interfaces.1 As a second-order nonlinear optical process, SFG is forbidden under the electric-dipole approximation in a medium with inversion symmetry but is necessarily allowed at a surface or interface where the inversion symmetry is broken. This unique feature makes SFG extremely sensitive to the surface or interfacial structure. Among many useful applications of SFG surface spectroscopy, studies of the interfacial structure of neat materials are of particular interest since few other techniques can yield the same information. Polymer surface structure is one such example. With Gabor Somorjai, we have initiated the study of polymer surfaces by SFG. We have been able to deduce from measurements surface composition, surface orientation of molecular groups, surface structure, and their response to the environment for a number of polymers.2–7

In interpreting the observed SFG spectra in reflection, one usually assumes that the spectra originate from the top surface monolayer. From a general physical argument, however, we can only conclude that the bulk electric-quadrupole contribution1,8 from a centrosymmetric medium is smaller than, or of the same order of magnitude as, the surface contribution to SFG in reflection.1,8 Without any prior knowledge about the surface structure, it is not clear whether the bulk contribution can always be neglected. To be certain, we need to know the surface and bulk contributions separately. This, in general, cannot be done.8–10 However, from SFG measurements in both reflection and transmission directions,11 we can obtain a good estimate of the bulk nonlinearity that allows us to judge whether it is negligible or not.

Section II discusses the theoretical background and assumptions behind our measurements. Section III presents the experimental scheme for measurements and comparison of reflected and transmitted SFG spectra. The technique has been applied to two different samples. The first one is a silane monolayer adsorbed on fused quartz (section IV) from which the reflected and transmitted SFG spectra appear to be well correlated as they should be. The other one is a polyethylene film (section V). There, the reflected and transmitted SFG spectra are significantly different, indicating that the bulk contribution is important for the transmitted SFG. We can then deduce the value of the bulk nonlinearity and show that its contribution to the reflected SFG is indeed negligible. In fact, the measured bulk nonlinearity also gives us an estimate of how weak the reflected SFG would have to be for the bulk contribution to be nonnegligible.

II. Theory

In a typical surface SFG experiment, two input laser beams at frequencies ω1 and ω2 overlap at a surface or interface to induce a surface nonlinear polarization

$$P_{S}^{(2)}(\sigma_s) = \epsilon_0 \chi_S^{(2)} \cdot \mathbf{E}(\omega_1) \mathbf{E}(\omega_2)$$ (1)

and generate a coherent SFG output at ωs in two well-defined directions (reflection and transmission, see Figure 1). Both directions are determined by the requirement that the parallel (λ) components of the input and output wave vectors are matched:

$$k_{s\lambda} + k_{2\lambda} = k_{s\lambda}$$ (2)

Scanning ωs over the frequency range of the surface molecular vibrational modes yields an SFG spectrum.

For a centrosymmetric bulk medium, although its nonlinear susceptibility χ0B vanishes under the electric-dipole approximation, it still has a bulk nonlinear polarization resulting from electric-quadrupole contributions1,8,10

$$P_{B}^{(2)}(\sigma_s) = \epsilon_0 \chi_B^{(2)} \cdot \mathbf{E}(\omega_1) \mathbf{E}(\omega_2) + \epsilon_0 \chi_B^{(2)} \cdot \mathbf{E}(\omega_1) \mathbf{E}(\omega_2) - \epsilon_0 \mathbf{V} \cdot [\chi_B^{(2)} \cdot \mathbf{E}(\omega_1) \mathbf{E}(\omega_2)]$$ (3)

This general form has also included the magnetic dipole contribution.12 If E(ω1) and E(ω2) can be treated as plane waves with wave vectors k1 and k2, then eq 3 becomes

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surface nonlinearity are of the same order as quadrupole in nature, and their contributions to the effective coherence length. Since $k_1 l_1$ effective surface nonlinear susceptibility 13.

The bulk contribution to SFG can be described in terms of an extra bulk contributions arising from rapid variations of the fields and the structure at an interface.10 They behave like a second-harmonic generation) measurements.9 There are also electric-quadrupole in nature, and their contributions to the effective surface nonlinearity are of the same order as $\chi^{(2)}$. Thus deduction of an order-of-magnitude value of $\chi^{(2)}$ from the measurement of $\chi_{ikb}^{SB}$ allows us to have an order-of-magnitude estimate of the importance of all the bulk contributions to SFG. In the following, we combine $\chi^{(2)}$ and the inseparable bulk contributions into one quantity $\chi^{(2)}_{SB}$ and express the total effective surface nonlinear susceptibility for SFG in the form.

Figure 1. Experimental setup for measuring SFG in (a) reflection and (b) transmission. M1, M2, and M3, metal mirrors to reflect the SFG signal; P1 and P2, iris diaphragms to direct the SFG and the guide He–Ne beams; D, photodetector; L1, lens to image the beam spot on the sample surface to diaphragm P2; L2, lens to image the beam spot on the sample surface back to itself with a slight parallel shift; W, flat window to shift the SFG beam. P1 and P2 in front of the detector. A He–Ne laser beam was then directed to trace the SFG beam path into detector D with the help of pinholes P1 and P2. With the real sample replacing the reference sample, the alignment was done by adjusting the sample position such that the reflected 1.064 μm input beam and the He–Ne laser beam retraced their previous beam paths.

In the actual SFG measurements, we had to separate reflected and transmitted SFG signals. This was accomplished by a small parallel shift of M3 to the right that displaced the transmitted SFG beam slightly from the reflected SFG beam on their way to the detector. Then only the reflected SFG output could go through the pinholes to reach the detector. To measure the transmitted SFG, we inserted flat window W in front of L1 to compensate for the shift of the transmitted SFG beam caused by the shift of M3 and, at the same time, shift the reflected SFG beam away. Only the transmitted SFG output could then reach the detector.

The above alignment procedure was found to be reliable and easy to implement. Since the reflected and transmitted SFG signals were recorded by the same detection system, the results were amenable to quantitative comparison.
IV. Study of Octadecyltrichlorosilane Monolayer

We have measured SFG spectra in both reflection and transmission from an octadecyltrichlorosilane (OTS) monolayer on a fused quartz substrate. The sample was prepared by the usual self-assembly technique. The thickness of the fused quartz substrate is about 3 mm, sufficient to eliminate the SFG contribution from the bottom surface. OTS has a long alkyl chain and is known to form a well-ordered self-assembled monolayer on glass. SFG spectra of the C–H stretch modes in reflection from such a monolayer were reported earlier. In this case, because the resonant SFG signal originated from a surface monolayer, the reflected and transmitted spectra were expected to be correlated.

The surface nonlinear susceptibility \( \chi_S^{(2)} \) of an OTS monolayer with \( C_{60} \) symmetry has only three independent nonvanishing elements:

\[
\begin{align*}
\chi_{S,zzz}^{(2)} &= \chi_{S,yyz}^{(2)} \\
\chi_{S,xzz}^{(2)} &= \chi_{S,xyz}^{(2)} \\
\chi_{S,xzz}^{(2)} &= \chi_{S,yzy}^{(2)} = \chi_{S,zxz}^{(2)}
\end{align*}
\]

In the last row, we have used the approximation \( \chi_{S,j,k}^{(2)} = \chi_{S,j,k}^{(2)} \) because \( \omega_1 \) and \( \omega_3 \) are far away from electronic resonances.

Near vibrational resonances, \( \chi_S^{(2)} \) takes the form

\[
\chi_S^{(2)}(\omega_2) = \chi_{NR}^{(2)} + \sum_q \frac{A_q}{\omega_2 - \omega_q + i \Gamma_q}
\]

where \( \chi_{NR}^{(2)} \) describes the nonresonant background and \( A_q, \omega_q, \) and \( \Gamma_q \) are the amplitude, resonant frequency, and damping constant, respectively, of the \( q \)th molecular vibrational mode.

In MKS units the SFG output intensity is given by

\[
I(\omega_2) = \frac{\omega_2^2}{8\epsilon_0^2 c^3 \beta_s} |\chi_S^{(2)}| ^2 I(\omega_1) I(\omega_2)
\]

where \( I(\omega_1) \) is the beam intensity at \( \omega_1 \) and \( |\chi_S^{(2)}| \) is the total effective surface nonlinear susceptibility defined as

\[
|\chi_S^{(2)}| = |\chi_{tot,eff}^{(2)} + \chi_{B,eff}^{(2)}|
\]

where \( \chi_{tot,eff}^{(2)} = [L(\omega_1) \cdot e_s] \cdot [L(\omega_2) \cdot e_1] |L(\omega_2) \cdot e_2| \)

with \( e \) being the unit polarization vector of the optical field at \( \omega_1 \) and \( L(\omega) \) being the tensorial Fresnel factor. In the present case, \( \chi_{B,eff}^{(2)} \) comes from the fused quartz substrate.

The Fresnel factors at the center of our tuning range have been calculated and listed in Table 2. Because the tuning range is relatively small (200 cm\(^{-1}\)) in this experiment, the dispersion of Fresnel factors due to the variation of \( \omega_2 \) can be neglected. Using these values, we can express all the surface effective nonlinear susceptibilities in terms of the three independent nonvanishing \( \chi_{S,j,k}^{(2)} \) elements:

\[
\begin{align*}
\chi_{S,eff}(\text{ssp}, R) &= 0.47 \chi_{S,zzz}^{(2)} \\
\chi_{S,eff}(\text{ssp}, T) &= 0.56 \chi_{S,zzz}^{(2)} \\
\chi_{S,eff}(\text{ppp}, R) &= 0.25 \chi_{S,zzz}^{(2)} - 0.42 \chi_{S,xxx}^{(2)} + 0.19 \chi_{S,zzz}^{(2)} \\
\chi_{S,eff}(\text{ppp}, T) &= 0.47 \chi_{S,zzz}^{(2)} - 0.42 \chi_{S,xxx}^{(2)} + 0.19 \chi_{S,zzz}^{(2)}
\end{align*}
\]

These equations show strict correlation between the reflected and transmitted SFG spectra from the surface contribution, in contrast to the effective bulk nonlinear susceptibility \( \chi_{B,eff}^{(2)} \), which can be strongly enhanced by the longer coherence length in the transmission direction.

Figure 2 shows the reflected and transmitted SFG spectra obtained from the OTS monolayer. To take into account the different losses of signal in reflection and transmission, all spectra were normalized against the SFG intensity from a z-cut crystalline quartz reference sample (see Appendix A for details), which makes it possible to plot all spectra in MKS units. The reflected and transmitted spectra in Figure 2 seem to be

| \( \lambda \) | 810 nm | 810 nm | 1064 nm | 3.4 \( \mu m \) |
| \( n \) | 1.453 | 1.453 | 1.450 | 1.410 |
| \( \beta \) | 20.0° | 20.0° | 45° | 57° |
| \( L_{xx} \) | 0.83 | 0.97 | 0.92 | 1.02 |
| \( L_{yy} \) | 0.80 | 0.96 | 0.72 | 0.65 |
| \( L_{zz} \) | 1.17 | 0.97 | 1.08 | 0.98 |
| \( L_{xx}e,(p) \) | -0.78 | 0.91 | 0.65 | -0.56 |
| \( L_{xx}e,(s) \) | 0.80 | 0.96 | 0.72 | 0.65 |
| \( L_{xx}e,(p) \) | 0.40 | 0.33 | 0.77 | 0.82 |

\( \omega_1(R) \) and \( \omega_1(T) \) represent reflection and transmission, respectively. \( L(\omega, T) \) includes the transmission coefficient of the bottom surface of the fused quartz substrate. The dielectric constant of the surface monolayer \( \varepsilon' = 1 \) was used for the calculation of \( L_{xx} \).
depends linearly on the wave vectors \( k \) contributing to the bulk term because, as shown by eq 5, this can be explained by the different nonresonant bulk contributions \( \chi_{\text{B,eff}}^{(2)} \) for the two different geometries. In fact, we were able to fit all the spectra (shown by the solid lines in Figure 2) using eqs 7 and 10 with the same set of \( A_q \), \( \Delta_{\nu} \), and \( \Gamma_\nu \) values and different nonresonant contributions \( \chi_{\text{tot,eff}}^{\text{NR}} \), as listed in Tables 3 and 4. The typical relative error of deduced \( A_q \) or \( \chi^{(2)} \) in SFG is about 10%.

The close fit in Figure 2 indicates that SFG spectra in reflection and transmission from the OTS monolayer are indeed very well correlated.

somewhat different, especially for the sps polarization combination (i.e., s-, p-, and s-polarized SFG output at \( \omega_3 \) and tunable infrared input at \( \omega_2 \), respectively), but this can be explained by the different nonresonant bulk contributions \( \chi_{\text{B,eff}}^{(2)} \) for the two different geometries. In fact, we were able to fit all the spectra (shown by the solid lines in Figure 2) using eqs 7 and 10 with the same set of \( A_q \), \( \Delta_{\nu} \), and \( \Gamma_\nu \) values and different nonresonant contributions \( \chi_{\text{tot,eff}}^{\text{NR}} \), as listed in Tables 3 and 4. The typical relative error of deduced \( A_q \) or \( \chi^{(2)} \) in SFG is about 10%.

The close fit in Figure 2 indicates that SFG spectra in reflection and transmission from the OTS monolayer are indeed very well correlated.

With the values of \( \chi_{\text{tot,eff}}^{\text{NR}} \) in both reflection and transmission, we could further deduce \( \chi_{\text{B,eff}}^{(2)} \). Here we take the sps polarization combination as an example. From eqs 6 and 9, we have

\[
\chi_{\text{B,eff}}^{(2)}(\text{sps}) = \frac{L_{yy}(\omega_1) e_{\omega_1} L_{zz}(\omega_1) e_{\omega_1}}{k_{zz} - k_{zz} - k_{zz}^2} \chi_{\text{tot,eff}}^{(2)}(\text{sps}) \chi_{\text{tot,eff}}^{(2)}(\text{sps})
\]

Here, we have two nonvanishing components \( \chi_{\text{B,eff}}^{(2)} \) and \( \chi_{\text{B,eff}}^{(2)} \) contributing to the bulk term because, as shown by eq 5, \( \chi_{\text{B,eff}}^{(2)} \) depends linearly on the wave vectors \( k' \) and \( k'' \), which break the inversion symmetry along the x- and z-axes. Equations 11 and 12 are valid for both resonant and nonresonant contributions.

Two only parameters are different for the reflection and transmission geometries. One is \( L_{yy}^{\text{s}}(\omega_1)e_{\omega_1} \), which can be found in Table 2, and the other is \( 1/k_{\omega_1} - k_{\omega_1} - k_{\omega_1}^2 \), which is equal to 49 and 720 nm for reflection and transmission, respectively. Inserting these values in eqs 11 and 12, we find

\[
\chi_{\text{tot,eff}}^{(2)}(\text{sps}) + \chi_{\text{B,eff}}^{(2)}(\text{sps}) = 0.52 \chi_{\text{tot,eff}}^{(2)}(\text{sps}) \chi_{\text{tot,eff}}^{(2)}(\text{sps}) + 49 \text{nm} \left[ \chi_{\text{B,eff}}^{(2)}(\text{sps}) \chi_{\text{tot,eff}}^{(2)}(\text{sps}) + \chi_{\text{B,eff}}^{(2)}(\text{sps}) \chi_{\text{tot,eff}}^{(2)}(\text{sps}) \right]
\]

\[
\chi_{\text{tot,eff}}^{(2)}(\text{sps}) + \chi_{\text{B,eff}}^{(2)}(\text{sps}) = 0.62 \chi_{\text{tot,eff}}^{(2)}(\text{sps}) \chi_{\text{tot,eff}}^{(2)}(\text{sps}) - 720 \text{nm} \left[ \chi_{\text{B,eff}}^{(2)}(\text{sps}) \chi_{\text{tot,eff}}^{(2)}(\text{sps}) + \chi_{\text{B,eff}}^{(2)}(\text{sps}) \chi_{\text{tot,eff}}^{(2)}(\text{sps}) \right]
\]

V. Study of Polyethylene Film

We have also measured the reflected and transmitted SFG in the C–H stretch region from a thin film of polyethylene on a fused quartz plate. The molecular structure of polyethylene is shown in the inset of Figure 3. The reflected SFG spectra were reported in an earlier study, and they were found to be dominated by the surface contribution. With transmitted SFG, we can determine the bulk contribution more quantitatively.

The polyethylene sample used in this experiment was prepared on a fused silica substrate using the following technique. A grain of low-density polyethylene was sandwiched between fused silica and sodium chloride windows. It was heated until it completely melted. Then it was squeezed between the two windows to form a film of 100–200 \( \mu \)m thick. The “sandwich” was then cooled to room temperature and immersed in distilled water. Owing to dissolution, the NaCl window was separated from the film, and a smooth polyethylene surface appeared, suitable for optical measurements. The film thickness was sufficient to absorb most of the infrared energy to prevent the polymer/silica interface from contributing to SFG. Therefore the SFG signal we measured must come from the air/polymer interface and the neighboring polymer bulk within the infrared absorption length.

Figure 3 displays the SFG spectra of polyethylene for the ssp and sps polarization combinations. While the reflection SFG spectra are essentially the same as those published in ref 2, the transmission SFG spectra are very different and can be explained...
only by the existence of the bulk contribution. The spectra show mainly three vibrational modes at 2850, 2884, and 2926 cm⁻¹. They can be assigned to the symmetric CH₂, Raman-active antisymmetric CH₂, and IR-active antisymmetric CH₂ stretch vibrations, respectively.²,¹⁸,¹⁹ One remarkable feature is that the Raman-active antisymmetric CH₂ stretch mode appears in the transmitted SFG spectra but not in the reflected SFG spectra. Being Raman-active and infrared-forbidden, this vibration mode can only be excited by the infrared field via electric-quadrupole excitation and therefore shows up only in the bulk contribution to SFG.

The significantly stronger SFG signal in the transmitted direction is due to a longer coherence length l_c that enhances the bulk contribution through the χ⁽²⁾B term in eq 6. From the measured spectra, we can obtain a rough estimate of χ⁽²⁾B. Here we consider the CH₂ symmetric stretch mode in the sps polarization combination, which appears to be the strongest peak in the transmitted SFG spectrum. Since polyethylene has a refractive index (~1.5) very close to that of fused quartz, we can still use eqs 13 and 14 as a good approximation because the Fresnel factors and the coherence length in eqs 11 and 12 are not very sensitive to the refractive index. For example, varying n from 1.45 to 1.60 only changes L_c(ωs,R) in Table 2 from 0.80 to 0.75.

From Figure 3 we find that at the peak of the CH₂ symmetric stretch mode

|χ⁽²⁾ tot,eff(sps,T)| ≈ 4 × 10⁻²¹ m² V⁻¹⁻¹
|χ⁽²⁾ tot,eff(sps,R)| ≈ 0

By solving eqs 13 and 14, we obtain the effective bulk contribution in the reflection direction

|χ⁽²⁾ B eff(sps,R)| ≈ 3 × 10⁻²² m² V⁻¹⁻¹

which is 1 order of magnitude smaller than the typical surface dipole contribution |χ⁽²⁾ tot,eff(R)| ≈ 10⁻²¹ m² V⁻¹⁻¹ on a vibrational resonance. This indicates that in the reflected SFG from polyethylene, the bulk contribution is indeed negligible.

The above value of |χ⁽²⁾ B eff(R)| suggests that with the reflection geometry the bulk contribution is usually negligible as long as the SFG signal is reasonably strong (i.e., |χ⁽²⁾ tot,eff(R)| > 3 × 10⁻²² m² V⁻¹⁻¹). This justifies the assumption in many cases that the reflected SFG spectra are dominated by surface contribution and can be used to probe surface structure.

VI. Conclusion

We have developed a scheme to measure both reflected and transmitted SFG spectra from the same sample and used them to estimate the relative contributions of surface and bulk to SFG. Measurements on an OTS monolayer adsorbed on fused quartz showed that the reflected and transmitted spectra originating from the monolayer are well correlated, while the nonresonant background resulting from the bulk contribution of the substrate is significant only in the transmitted spectra. That the bulk contribution is important only for transmitted SFG is also true in the case of polyethylene. A resonant mode that is only allowed in the bulk was found to appear only in the transmitted SFG spectra. Even though the bulk contribution is significant and easily detected in the transmitted SFG, our results suggest that the bulk contribution to the reflected SFG spectra is usually negligible.

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Appendix A. Effective Surface Nonlinear Susceptibility of Crystalline Quartz

Equation 8 shows that we can measure |χ⁽²⁾ eff|² of a sample by comparing its SFG intensity with that from a standard reference sample with a known χ⁽²⁾ eff. In this experiment, we used a z-cut α-SiO₂ (quartz) crystal as our reference sample. The SFG signal from crystalline quartz is mainly from the bulk because it does not have inversion symmetry. Quartz crystal has D₃ symmetry with the following nonvanishing χ⁽²⁾ijk elements

χ⁽²⁾ ixx = −χ⁽²⁾ iyy = −χ⁽²⁾ iyy = −χ⁽²⁾ ixx
χ⁽²⁾ ixy = −χ⁽²⁾ ixy
χ⁽²⁾ iyy = −χ⁽²⁾ iyy
χ⁽²⁾ ixy = −χ⁽²⁾ ixy
χ⁽²⁾ iyy = −χ⁽²⁾ iyy

among which χ⁽²⁾ ixx (defined as χ⁽²⁾ q below) and those equal to χ⁽²⁾ ixx are much larger than the others.²⁰ In the following calculation we neglect the weaker ones. For the z-cut quartz crystal, the SFG intensity is maximized when the x-axis of the crystal is in the laser incidence plane. With this geometry, the absolute values of the effective surface nonlinear susceptibilities defined by eq 9 are

|χ⁽²⁾ eff(ssp)| = g cos β₁ L_c(ω₁) L_c(ω₂) χ⁽²⁾ q ixx l_c
|χ⁽²⁾ eff(sps)| = g cos β₁ L_c(ω₁) L_c(ω₂) χ⁽²⁾ q ixy l_c
|χ⁽²⁾ eff(ppo)| = g cos β₁ cos β₂ L_c(ω₁) L_c(ω₂) χ⁽²⁾ q ixx l_c

(A1)

Here, β and L_c are the incidence angle and Fresnel factors, both listed in Table 5. g = 2 is a degeneracy constant that arises from the number of distinguishable permutations of the input frequencies.²⁰ l_c is the SFG coherence length, which, in our case, is approximately 46 and 755 nm for the reflection and transmission geometries, respectively.

Since in our experiment the SFG from crystalline quartz is nonresonant, we may neglect the dispersion and assume that

|Table 5: Parameters Used to Calculate |χ⁽²⁾ eff|² of a z-Cut Quartz Crystal²⁰

<table>
<thead>
<tr>
<th>ω₁(R)</th>
<th>ω₁(T)</th>
<th>ω₂</th>
<th>λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.534</td>
<td>1.534</td>
<td>1.49</td>
<td></td>
</tr>
<tr>
<td>20.0°</td>
<td>20.0°</td>
<td>45°</td>
<td>57°</td>
</tr>
</tbody>
</table>

² For convenience the birefringence (and also the optical activity) of the crystal is neglected and the refractive index of the ordinary wave n₀ is used for all polarizations.
\[ Z_{eq}^{(2)} = 2d_{11} \approx 8.0 \times 10^{-13} \text{m} \left( \frac{\text{V}}{\text{A}} \right) \]  
(A2)

where \( d_{11} \) refers to the nonlinear coefficient for second harmonic generation and its value for \( \lambda = 1.064 \mu \text{m} \) found in ref 20 was used.

We then find from eqs A1

\[
\begin{align*}
|Z_{\text{eff}}^{(2)}(\text{ssp,R})| &= 2.1 \times 10^{-19} \text{m}^2 \left( \frac{\text{V}}{\text{A}} \right) \\
|Z_{\text{eff}}^{(2)}(\text{sp,R})| &= 2.2 \times 10^{-20} \text{m}^2 \left( \frac{\text{V}}{\text{A}} \right) \\
|Z_{\text{eff}}^{(2)}(\text{pp,R})| &= 2.0 \times 10^{-20} \text{m}^2 \left( \frac{\text{V}}{\text{A}} \right) \\
|Z_{\text{eff}}^{(2)}(\text{ssp,T})| &= 4.3 \times 10^{-19} \text{m}^2 \left( \frac{\text{V}}{\text{A}} \right) \\
|Z_{\text{eff}}^{(2)}(\text{sp,T})| &= 4.5 \times 10^{-19} \text{m}^2 \left( \frac{\text{V}}{\text{A}} \right) \\
|Z_{\text{eff}}^{(2)}(\text{pp,T})| &= 3.8 \times 10^{-19} \text{m}^2 \left( \frac{\text{V}}{\text{A}} \right)
\end{align*}
\]  
(A3)

References and Notes


(13) See, for example: Shen, Y. R. The Principles of Nonlinear Optics; Wiley: New York, 1984; Chapter 25.


(17) If the intensities \( I(\omega_0), I(\omega_1), \) and \( I(\omega_2) \) are defined in a medium with a dielectric function \( \epsilon(\omega_0) \), eq 8 should be modified to (see, for example, ref 1).

\[
I(\omega_2) = \frac{\alpha_{2}^{(2)}|\beta(\omega_1)|I(\omega_1)I(\omega_2)}{8\epsilon_0 c^3 \cos^2 \beta \sqrt{\epsilon(\omega_1)\epsilon(\omega_2)}}
\]

Therefore, eq 8 is only valid with \( \epsilon(\omega_0) = 1 \), which is the case for vacuum or air. For the transmitted SFG signal, eq 8 is still valid if we define \( K(\omega_0) \) as the intensity of the SFG beam in air after it propagates through the bottom surface of the fused quartz substrate. In this case the transmission coefficient at the bottom surface must be included properly to calculate the Fresnel factor \( L(\omega_0) \) for the transmission geometry.

